



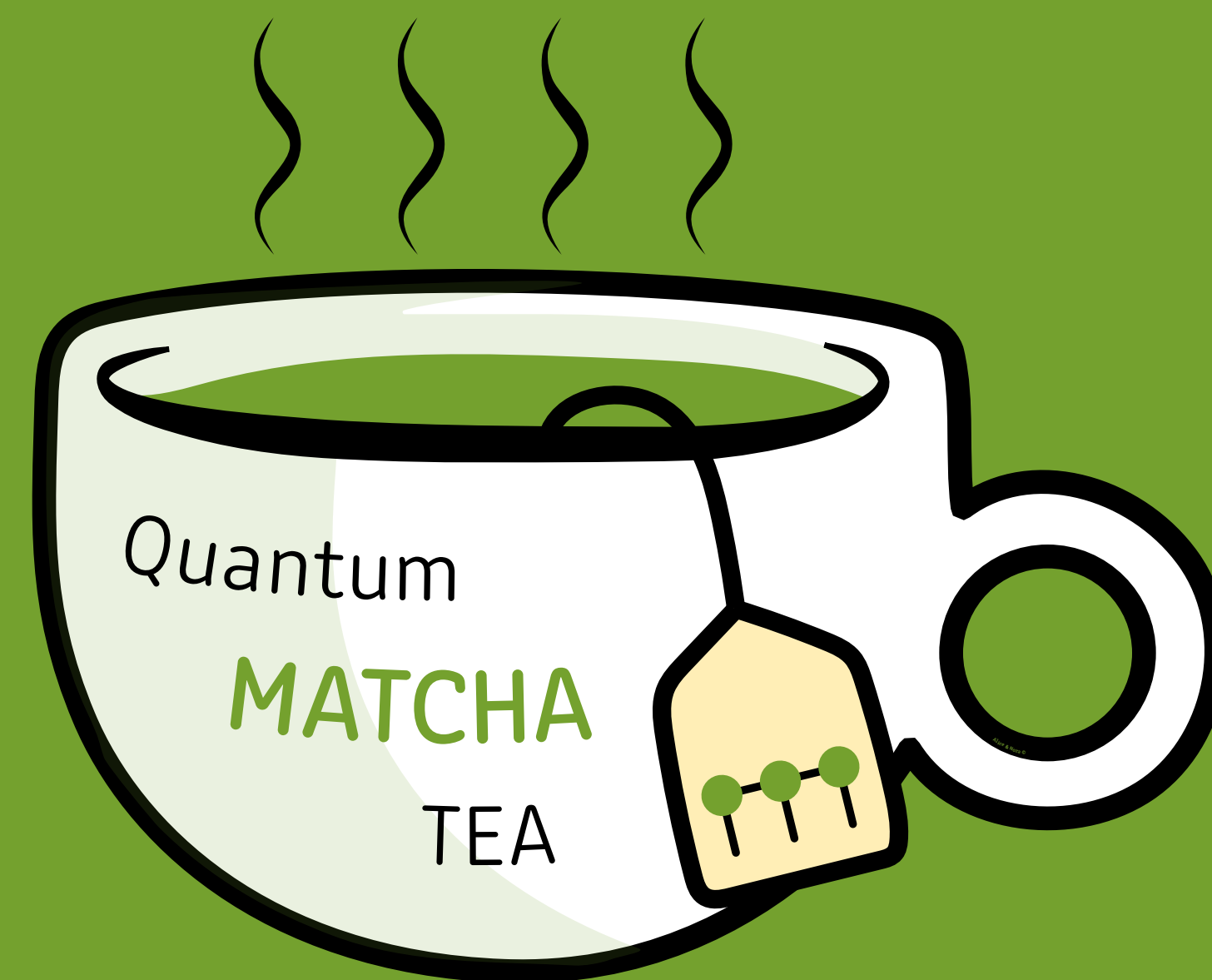
QUANTUM
COMPUTING
AND
SIMULATION
CENTER

QCSC Lecture
March 21 2023

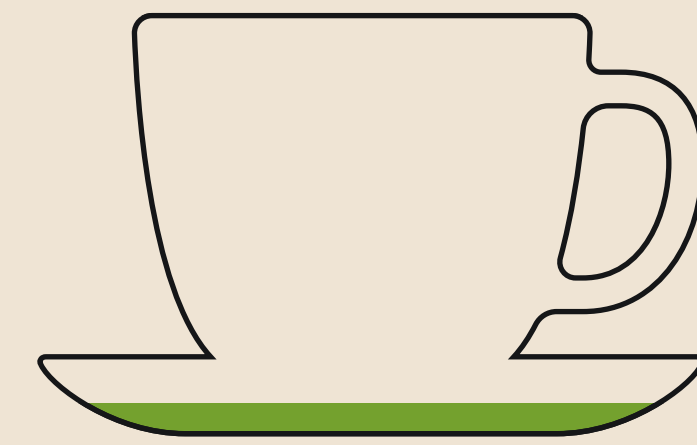
Introduzione all'emulatore di calcolatore quantistico HPC

Quantum Matcha Tea

Marco Ballarin
Università degli studi di Padova

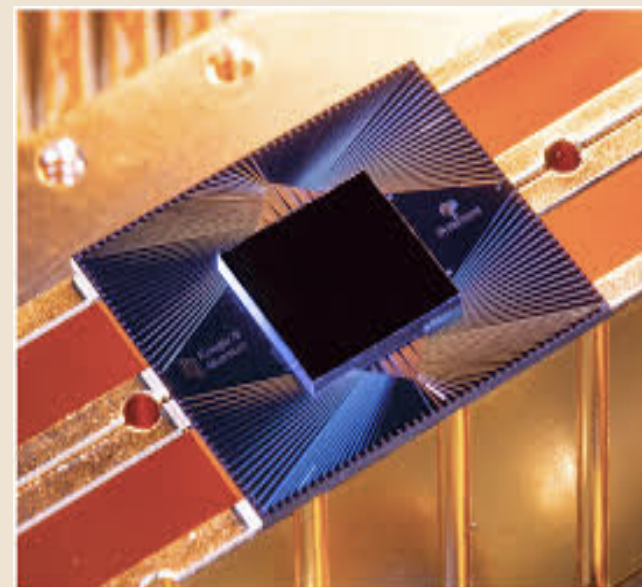


Running quantum algorithms

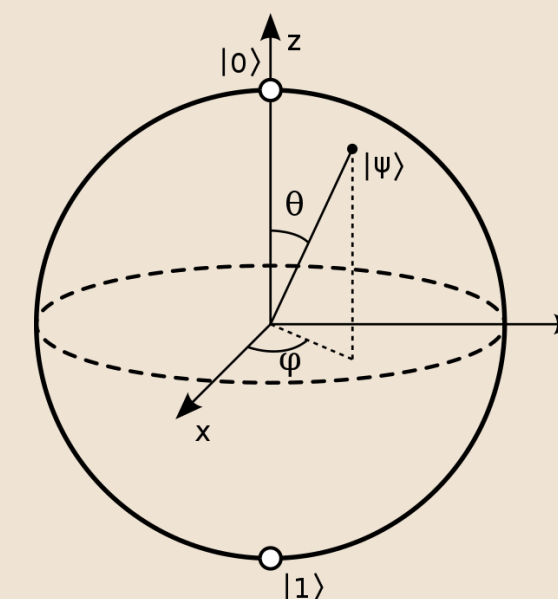
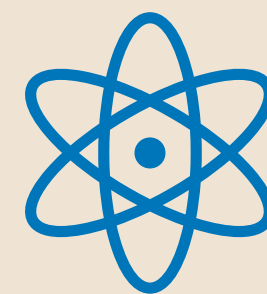


Also algorithms prepared for a quantum simulator can be digitised in quantum circuits

Quantum algorithm



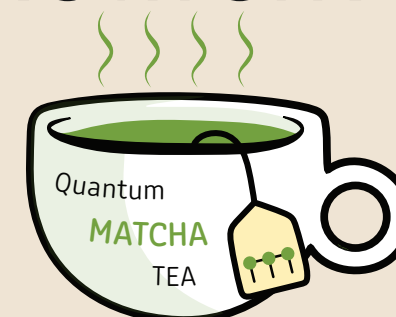
- + Real hardware
- Noisy
- Limited number of qubits



- + Access to exact state
- Limited # of qubits

Exact simulator

Tensor Network simulator

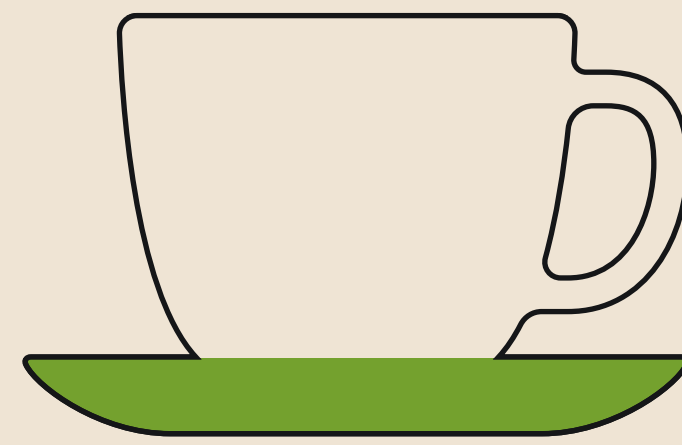


- + High # of qubits
- Limited entanglement



Quantum hardware

Quantum computation recap



QUBIT

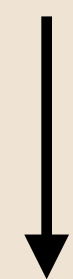
Fundamental unit of quantum information

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \alpha, \beta \in \mathbb{C}, \quad |\alpha|^2 + |\beta|^2 = 1$$

Two qubit states

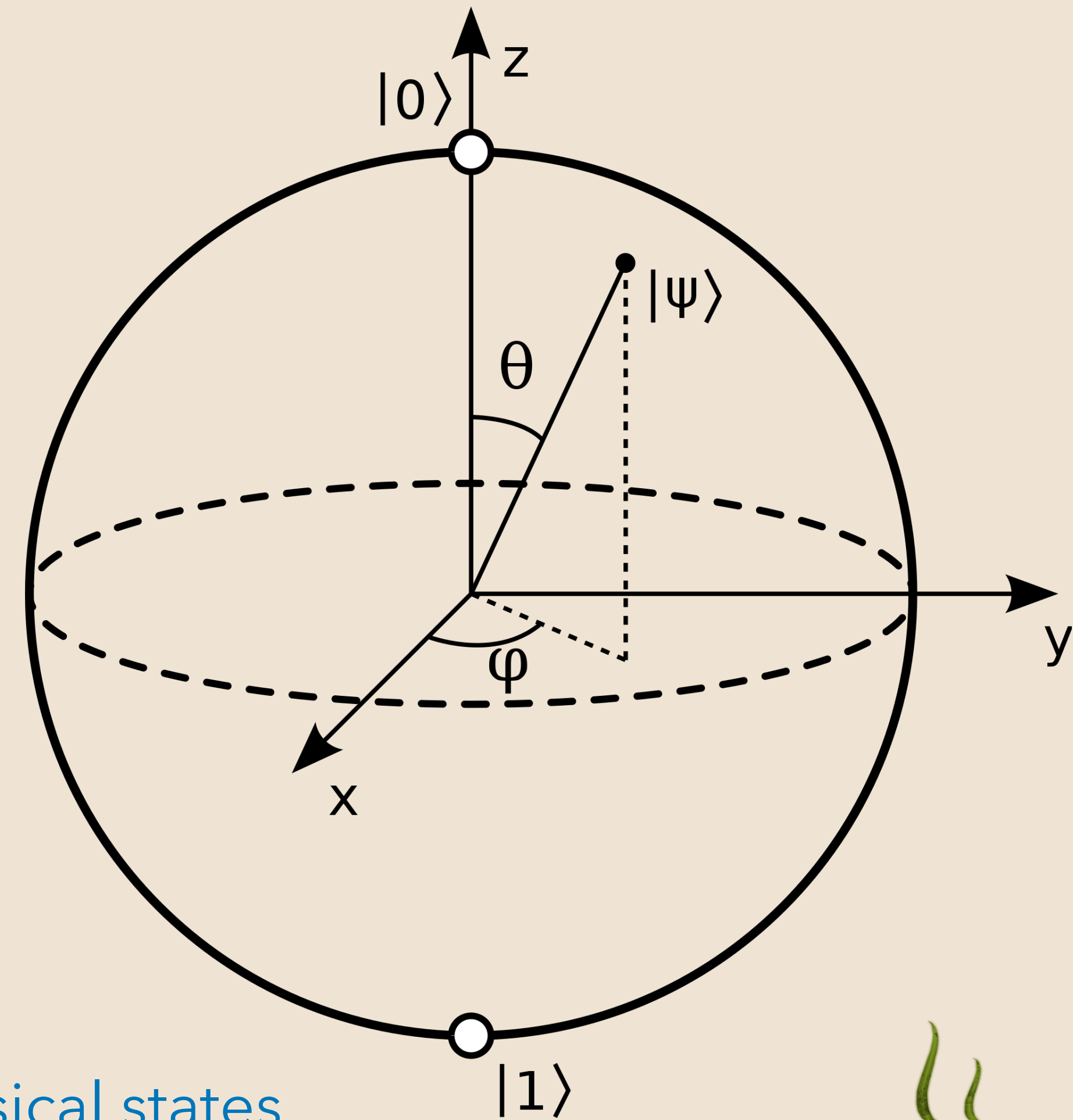
Number of coefficients scales as 2^n , n number of qubits

$$|\psi\rangle = |\underline{\alpha}|00\rangle + |\underline{\beta}|01\rangle + |\underline{\gamma}|10\rangle + |\underline{\eta}|11\rangle$$

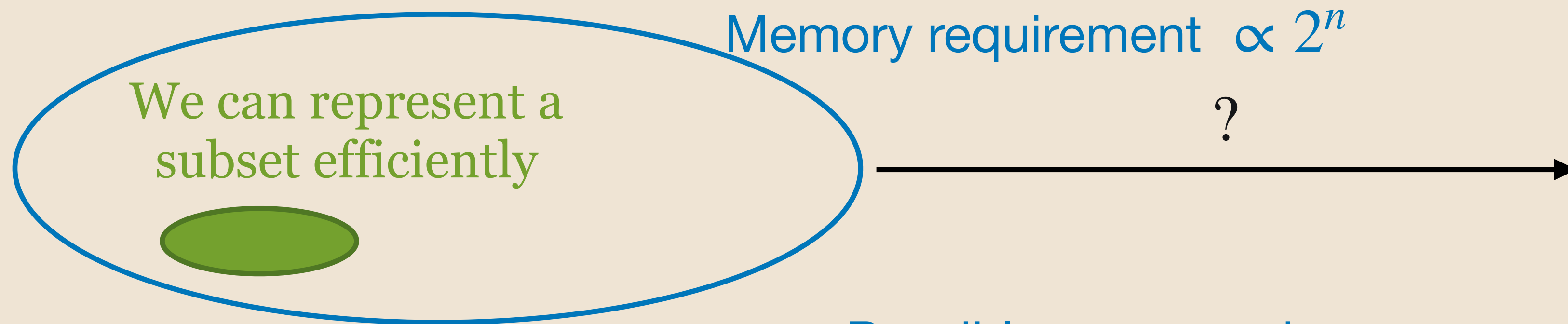
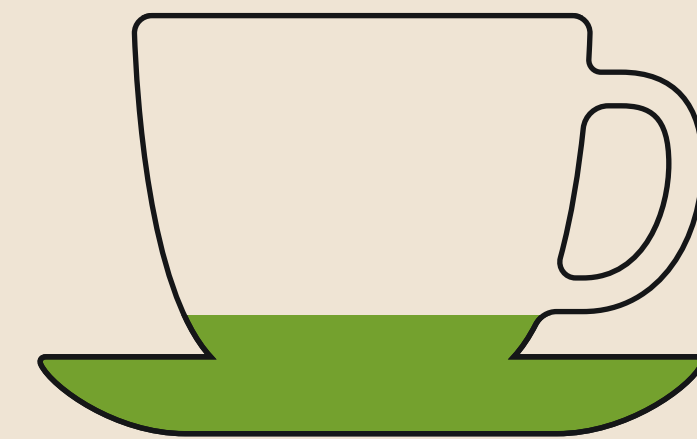


Entangled states

$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle \quad \text{Cannot be written as classical states} \\ \Rightarrow \text{display quantum correlation}$$



Entanglement and compression



Possible compression
quantified by Shannon entropy

Classical
bit string

0000

Optimal
Compression

0011

Here we can compress something.

The quantum state is **entangled**, it
cannot be written as classical state

RANDOM

No compression
possible without
approximations

Quantum
state

$|0000\rangle$

$$\frac{1}{\sqrt{2}} (|0000\rangle + |1111\rangle)$$

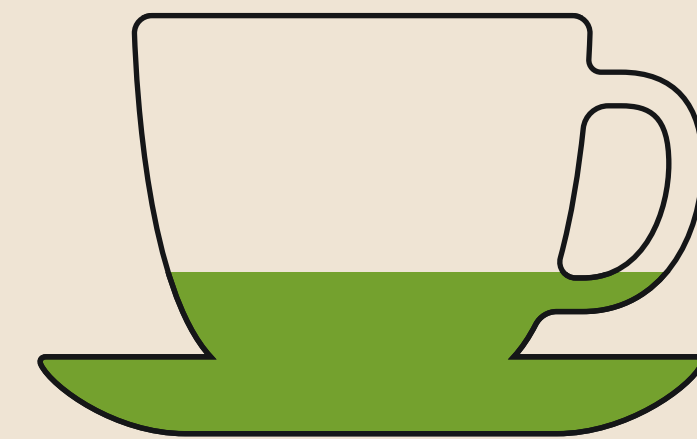
$|\text{RANDOM}\rangle$

Possible compression

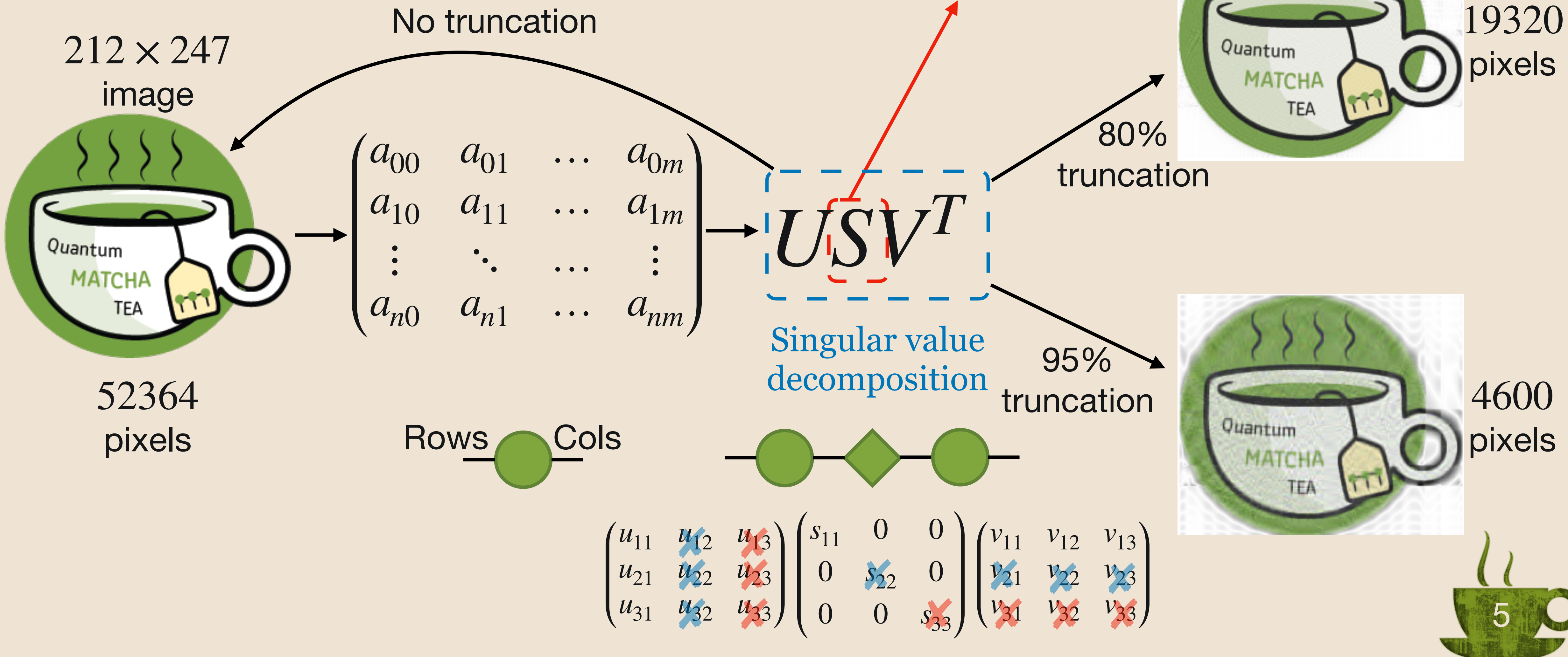
quantified by Von Neumann **entanglement** entropy



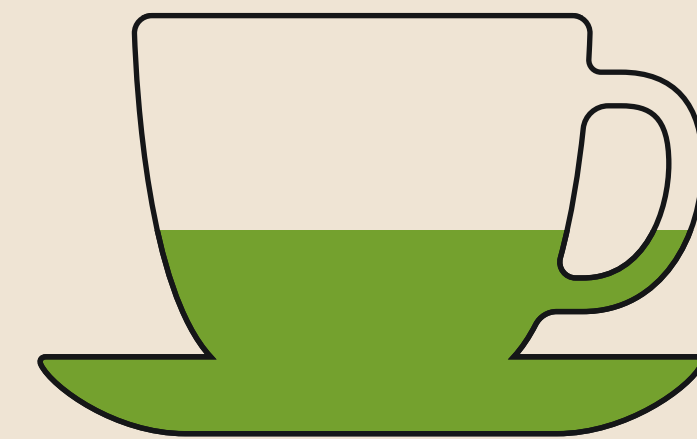
Image compression through SVD

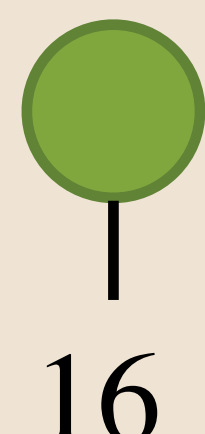
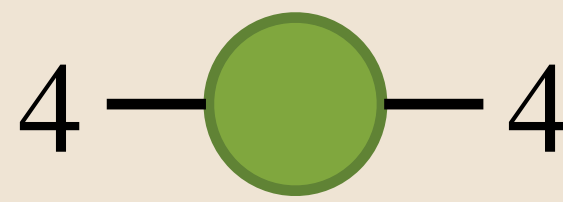


Truncate the "least important" singular values using the truncated norm

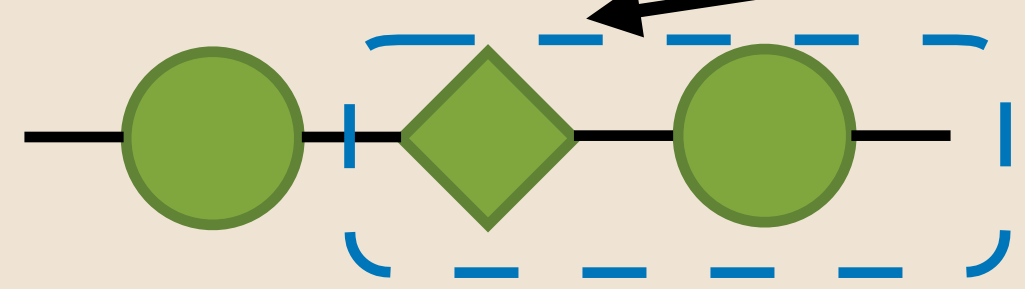


Compressing a 4-qubits state



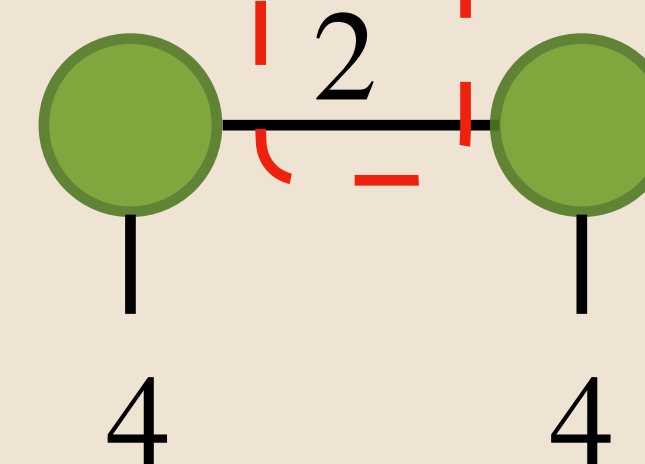

 $|\psi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$


Bond dimension encode entanglement between qubits

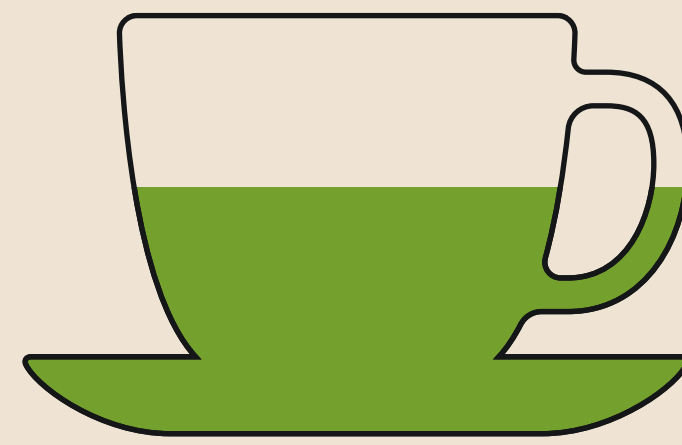

Matrix multiplication

$$\begin{pmatrix} 1 & 0 & \cancel{0} & \cancel{0} \\ 0 & 0 & \cancel{1} & \cancel{0} \\ 0 & 0 & \cancel{0} & \cancel{1} \\ 0 & 1 & \cancel{0} & \cancel{0} \end{pmatrix}
 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cancel{0} & 0 \\ 0 & 0 & 0 & \cancel{0} \end{pmatrix}
 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \cancel{0} & \cancel{1} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \end{pmatrix}$$

$4 \rightarrow 2$

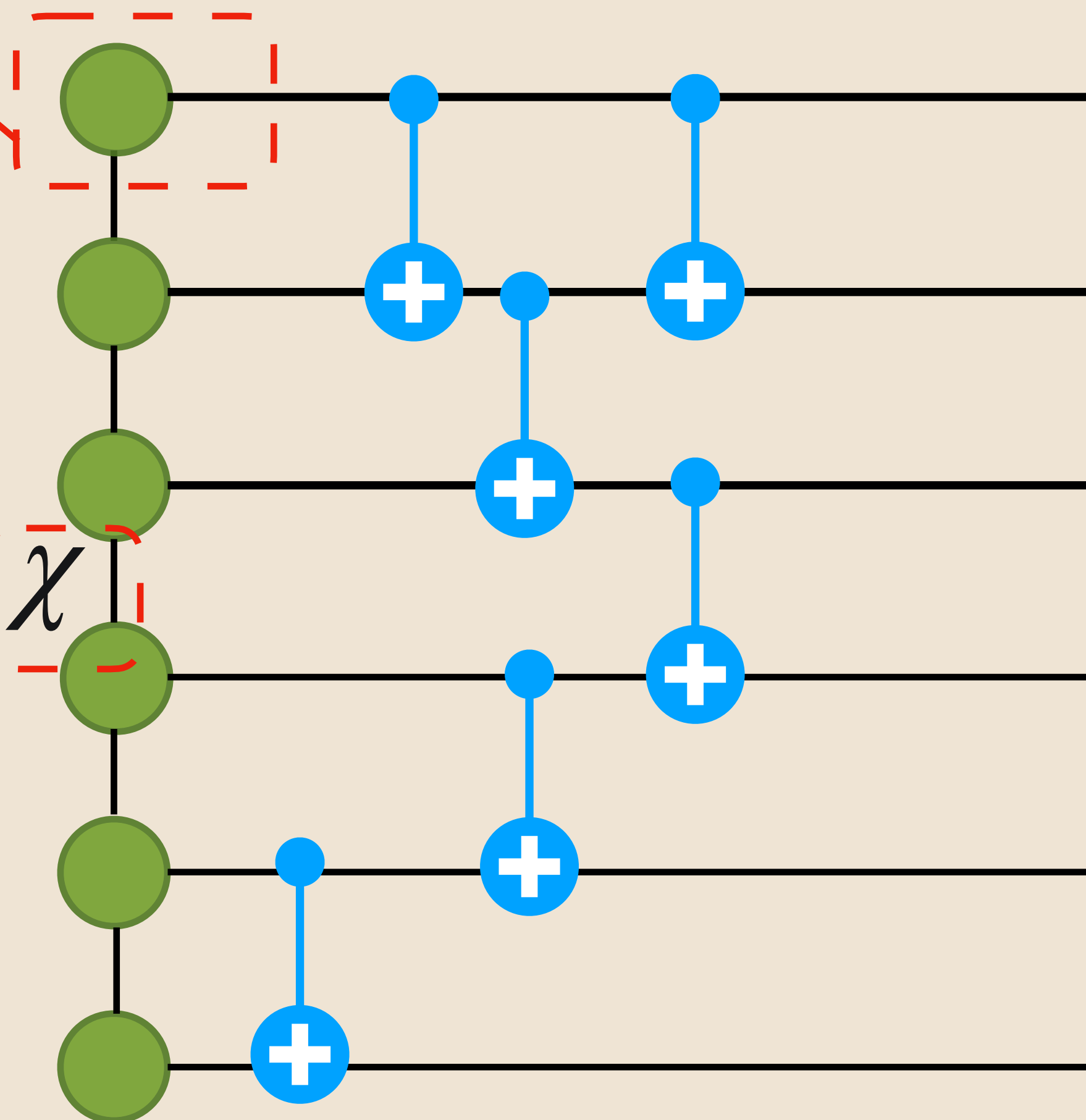


Matrix product states



Each tensor (ball) encodes the state of a qubit

Bonds encode entanglement between qubits



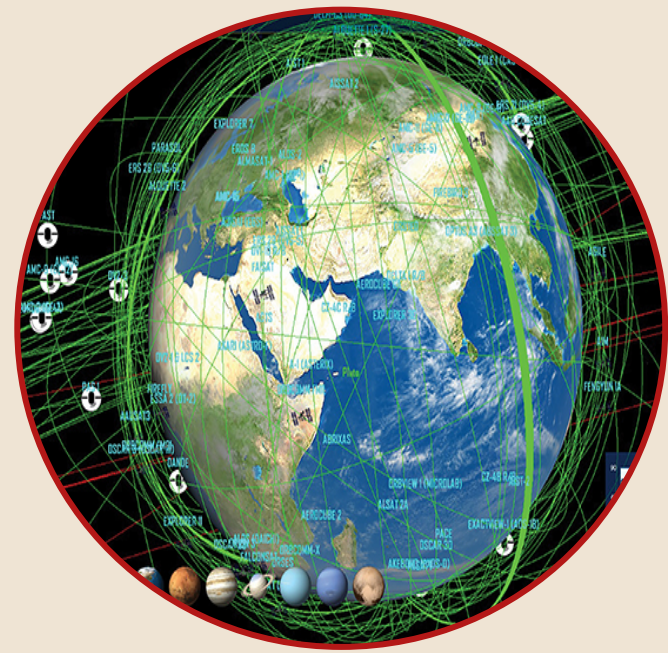
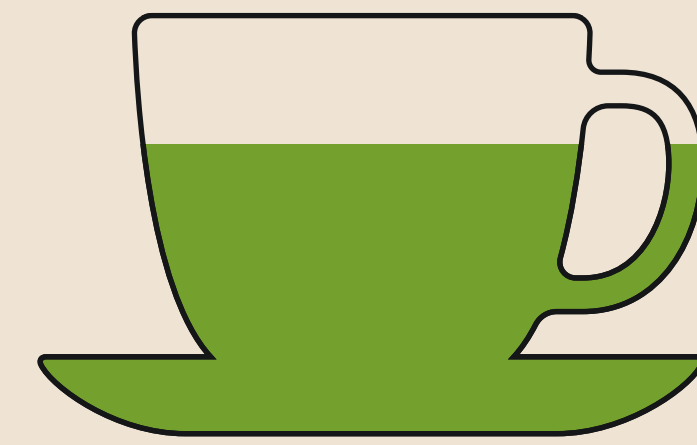
Memory requirements

$$O(2^n) \rightarrow O(2n\chi^2)$$

MPS SIMULATIONS ARE NOT LIMITED BY THE NUMBER OF QUBITS BUT BY THE ENTANGLEMENT



Quantum algorithms

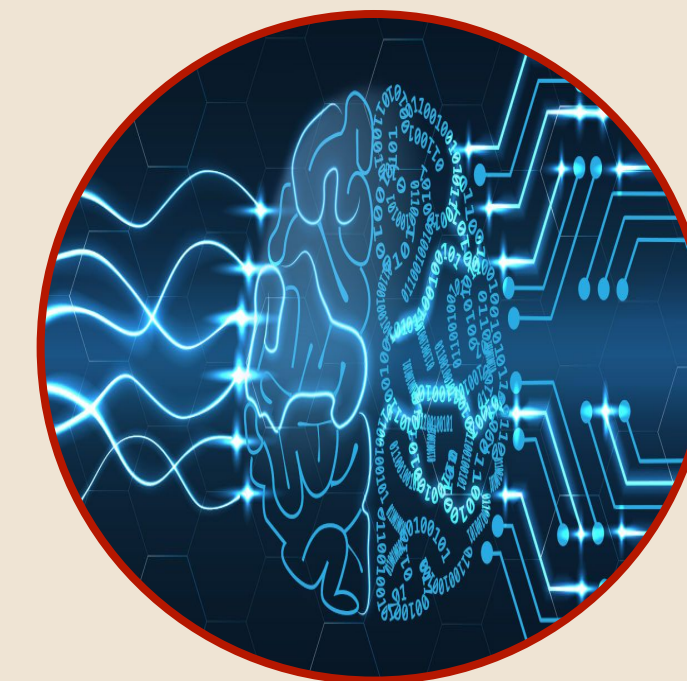
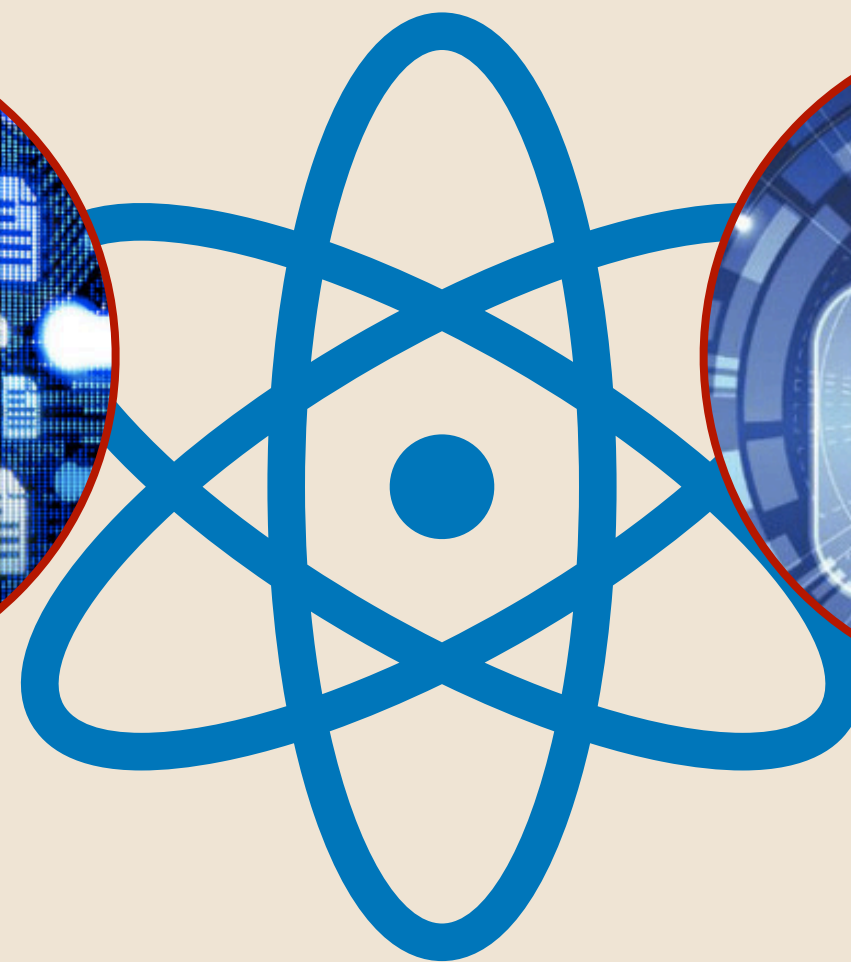


Earth Observation

Grover algorithm



Shor algorithm



Machine Learning

Combinatorial optimization problems

(QAOA, quantum annealing, ...)

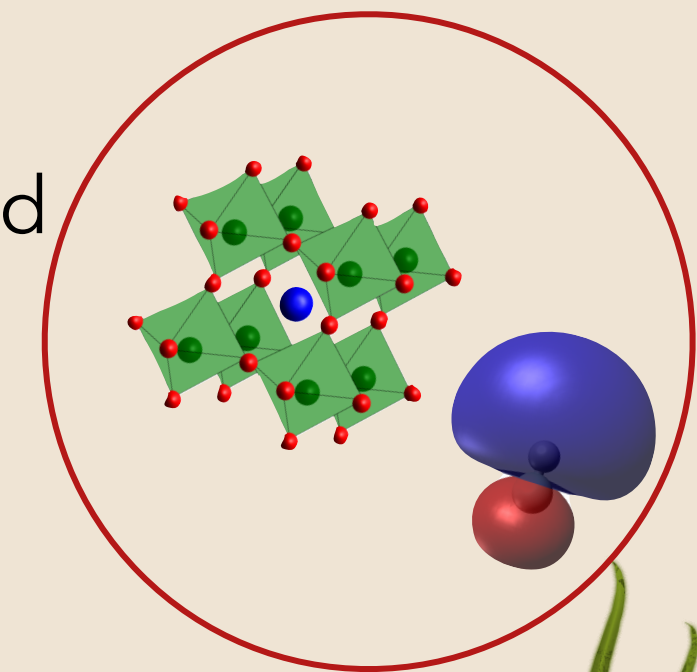


Traffic

Portfolio optimization



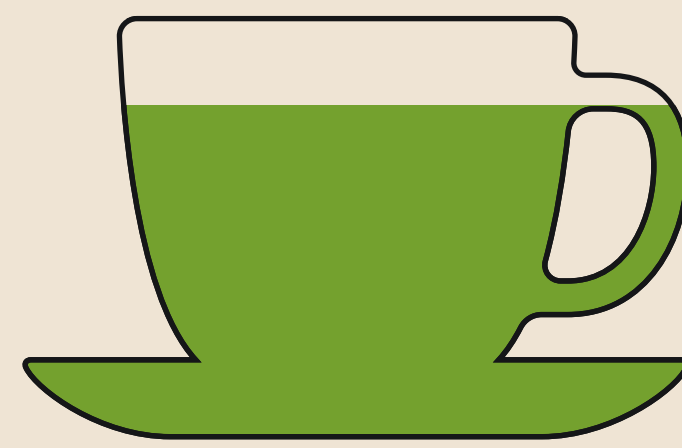
Molecules and Materials



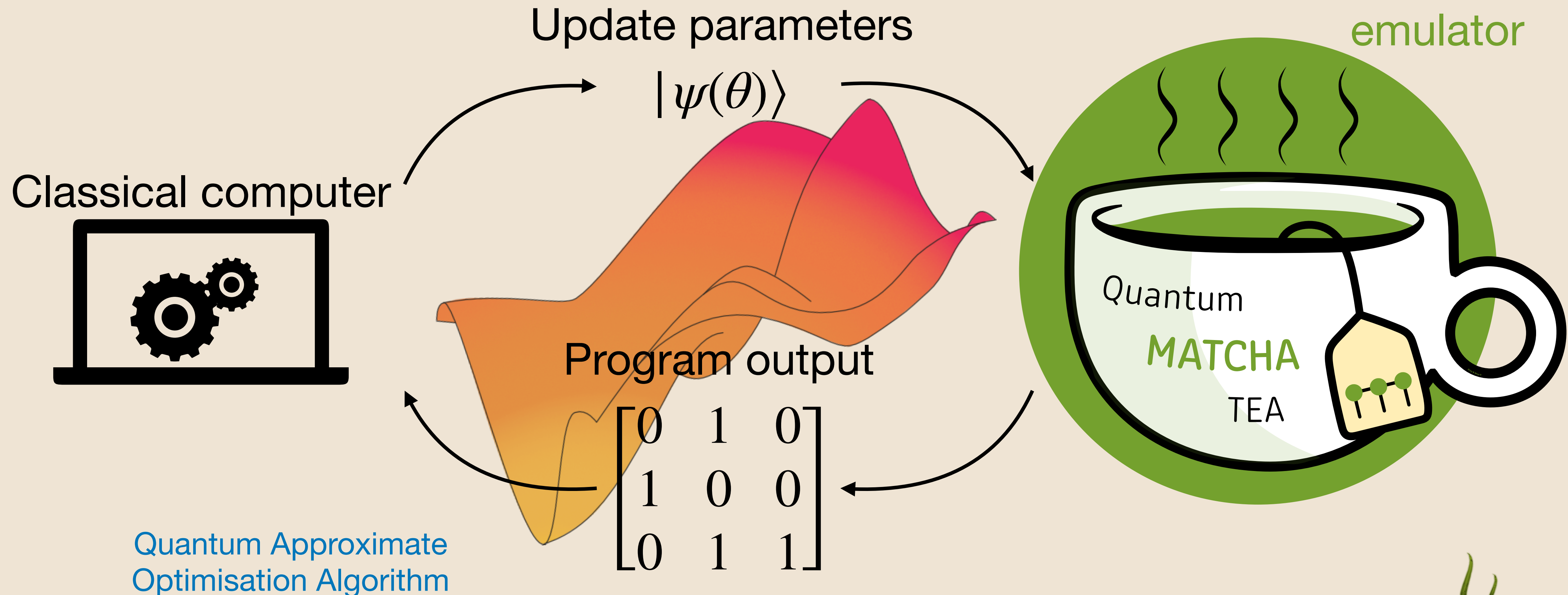
(VQE, quantum deflation, ...)



Hybrid quantum optimisation algorithms



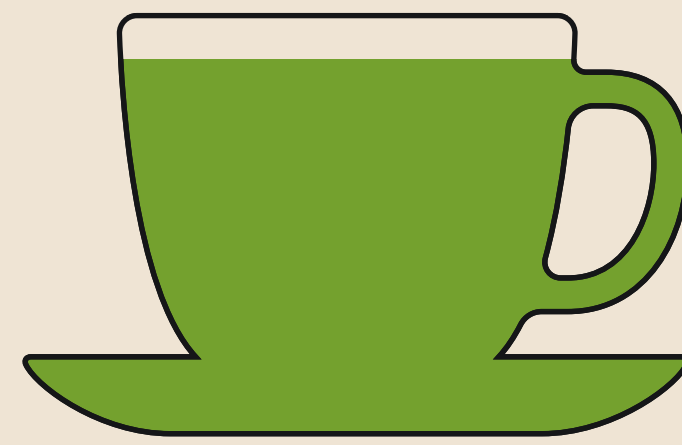
- **Objective:** minimize a cost function $f(\theta)$



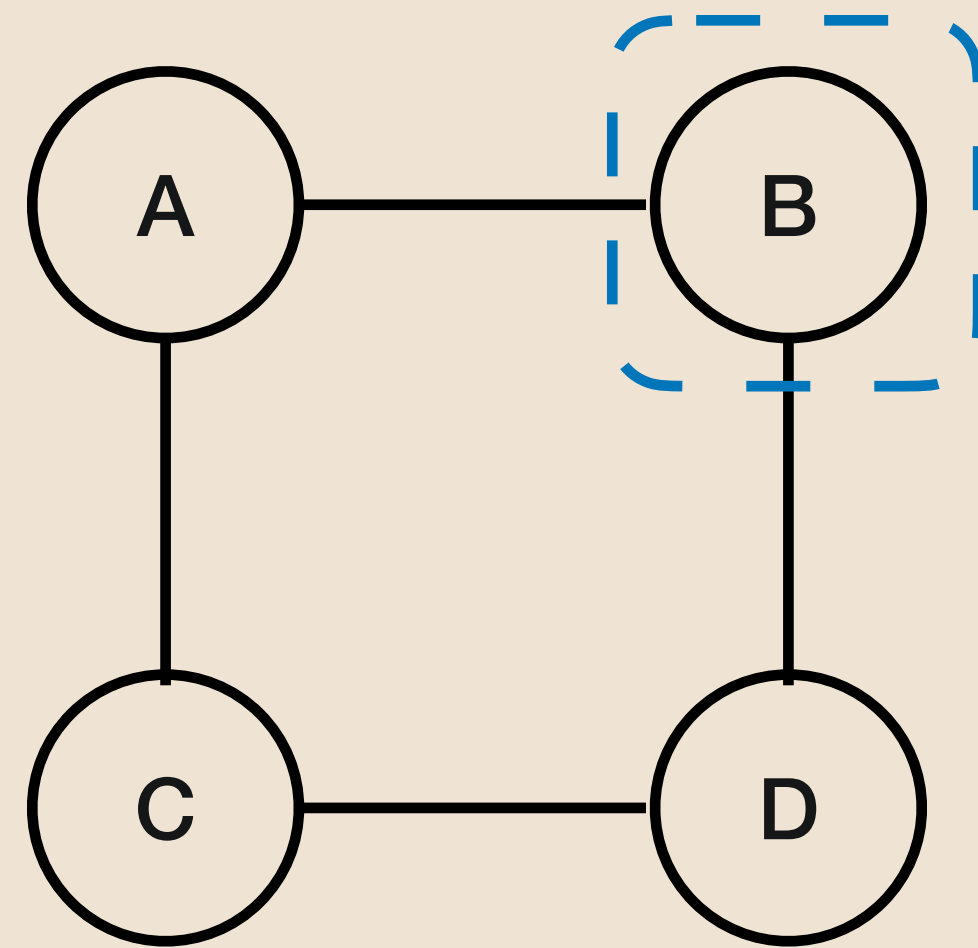
- **Example:** QAOA, inspired by Quantum Annealing (see DWave lecture)



Graph colouring problem



- Assign a color to each node
- Connected nodes should have different colours



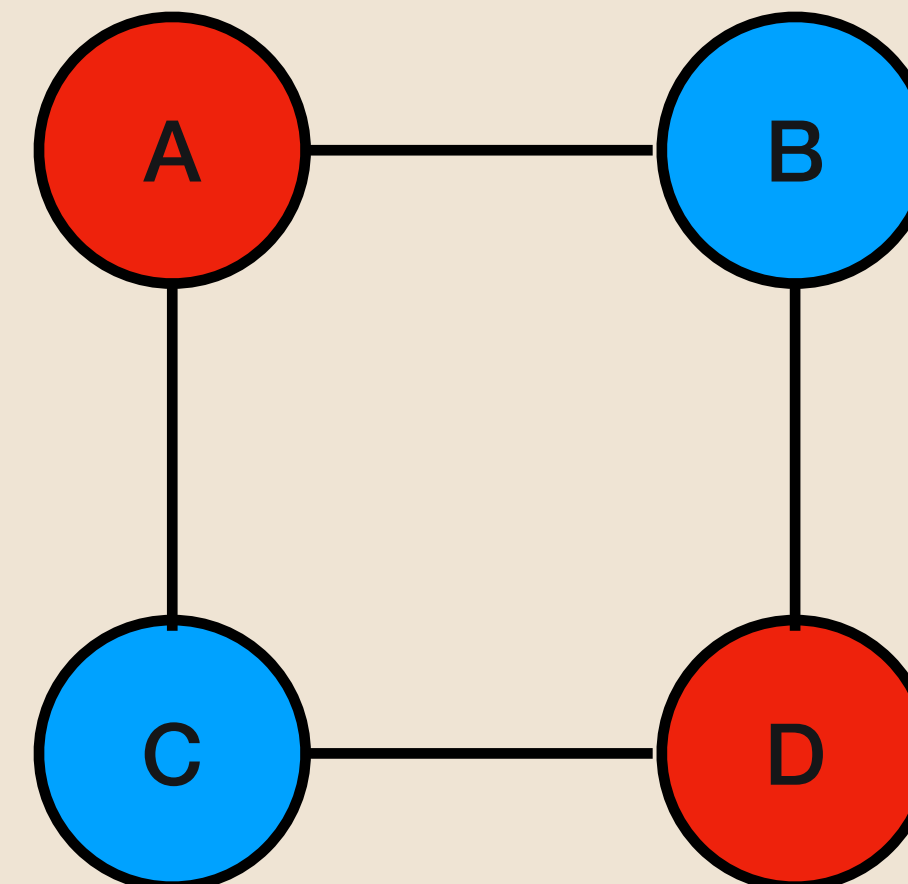
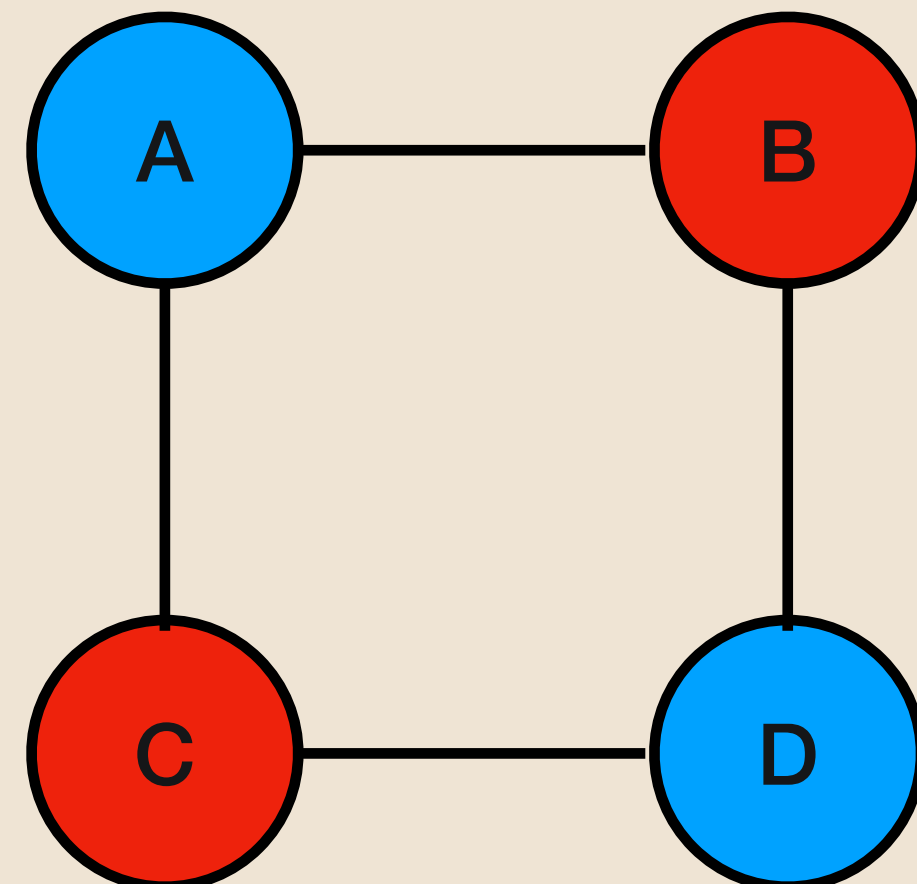
Each node is represented with binary variables:

- $|10\rangle$ blue
 - $|01\rangle$ red
- (One hot encoding)

Example task:
colouring maps



Solutions

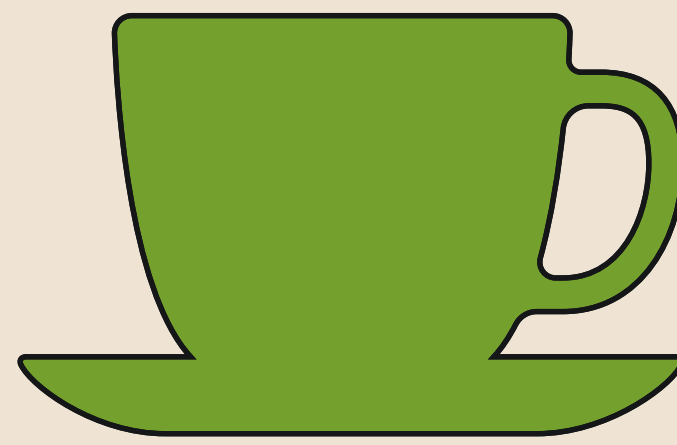
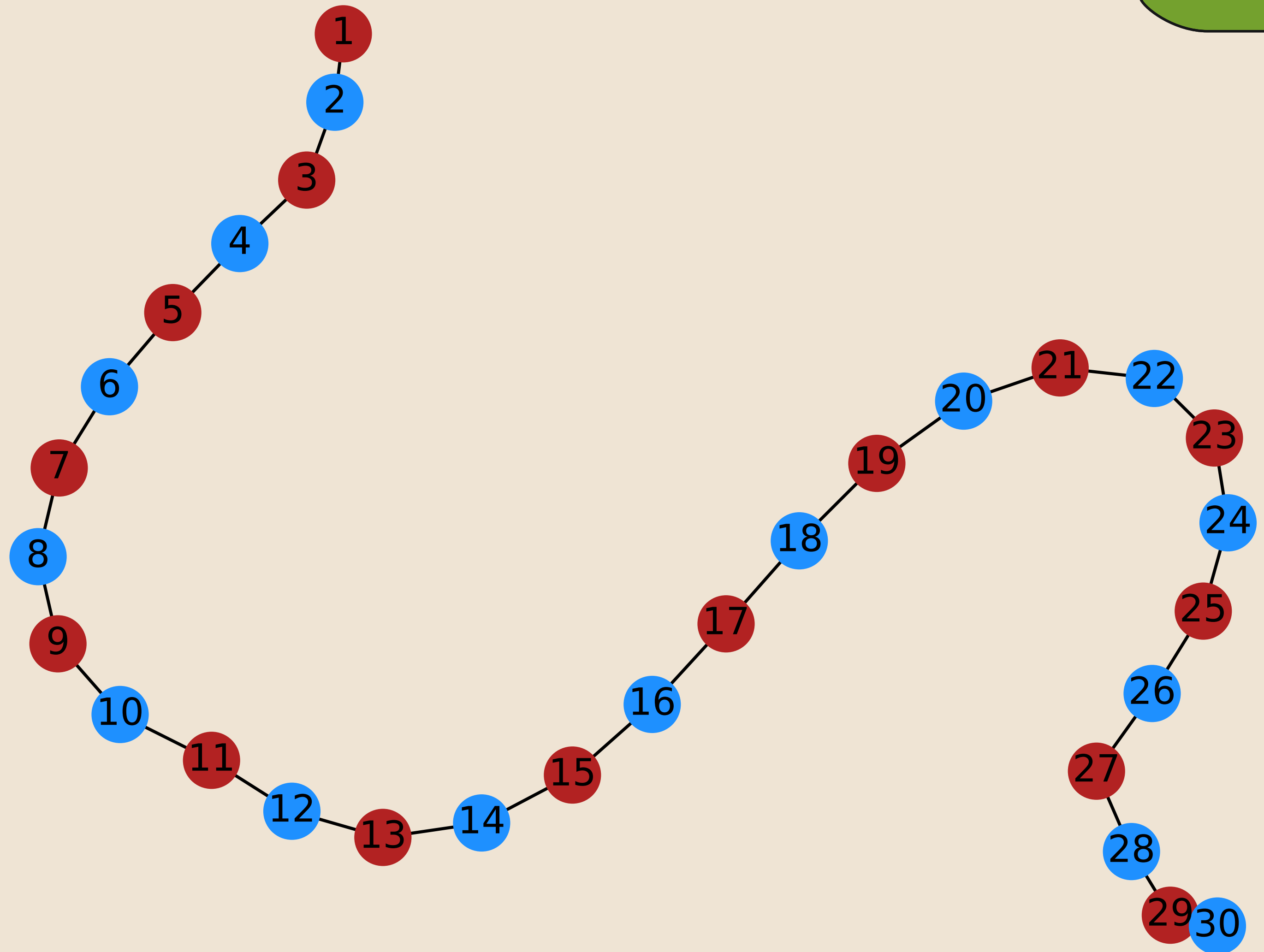


Conclusions

MPS simulations are not limited by the number of qubits but by the entanglement

Easy-to-use python frontend and fast HPC-ready backend (Both GPU and CPU)

Error analysis tools and efficient computations of observables optimised for the MPS representation



Thanks for your attention



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di Fisica
e Astronomia
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WORLD QUANTUM DAY

IN PADOVA

14 APRILE 2023

CENTRO UNIVERSITARIO
VIA ZABARELLA 82



PROGRAMMA

DALLE 14:00
FROM
ALLE 19:30
TO

GIOCHI DA TAVOLO
BOARD GAMES

VIDEOGIOCHI
COMPUTER GAMES

LABORATORIO D'ARTE
ART LAB

...AND MORE

INGRESSO LIBERO
FREE ENTRANCE

EDIZIONE
1

JOIN THE
QUANTUM
SIDE!

